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Journal of Econometrics 126 (2005) 1–24
JOURNAL OF Econometrics
www.elsevier.com/locate/econbase

Testing for common deterministic trend slopes☆

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Accepted 9 February 2004

Abstract

We propose tests for hypotheses on the parameters for deterministic trends. The model framework assumes a multivariate structure for trend-stationary time series variables. We derive the asymptotic theory and provide some relevant critical values. Monte Carlo simulations suggest which tests are more useful in practice than others. We apply our tests to examine real GDP convergence for a sample of seven European countries.

JEL classification: C12; C32

Keywords: Deterministic trends; Hypothesis testing; Multivariate trend function testing; Economic convergence

1. Introduction

In several empirical situations it is found that time series data contain a deterministic trend, while they are otherwise stationary. An example in macroeconomics concerns differences between real output for pairs of countries (see Hobijn and Franses (2000) among others) or pairs of regions within the U.S. (see Carlino and Mills (1993), Loewy and Papell (1995) and Tomljanovich and Vogelsang (2002) among others), where such a trend-stationary series indicates some degree of convergence. Other examples can be found in disciplines such as tourism and marketing, where tourist arrivals and sales

☆ The computer programs used for all calculations in this paper can be obtained from the corresponding author. Additional simulation results can also be obtained upon request. Work on this paper was initiated when the first author visited the Tinbergen Institute Rotterdam (August 1998) and the second author visited Cornell University (May 1999).

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0304-4076/$ - see front matter
doi:10.1016/j.jeconom.2004.02.004

often display upward trending patterns. Finally, environmental data like temperatures may also display trends, and if these are upward moving this can be taken as evidence of global warming, see Bloomfield (1992), Woodward and Gray (1993), Zheng and Basher (1999) and Fomby and Vogelsang (2002) among many others.

In some of the above cases it may be of interest to examine if two or more trend-stationary time series have the same slope. This would allow for testing whether a pair of countries are converging with the same speed as another pair. In the empirical portion of this paper, we ask whether gross domestic product (GDP) growth in Italy has been faster than in other European countries. Are the differences in growth between Italy and other countries the same? These are just some examples of empirically interesting tests that involve joint restrictions on the slopes of trend functions of multiple time series. While there has been recent research on univariate trend function inference and modeling (see Perron, 1991; Canjels and Watson, 1997; Vogelsang, 1997, 1998), multivariate trend modeling and inference has received little attention.[1] The goal of this paper is to propose and analyze tests regarding the slopes of multiple trend-stationary time series including null hypotheses that involve linear cross equation restrictions.

The outline of our paper is as follows. In Section 2, we discuss the model representation, parameter estimation, and the test statistics of interest. A key issue is the estimation of the asymptotic covariance matrix, for which we aim to compare three different approaches, amongst which is the familiar heteroskedasticity autocorrelation consistent (HAC) estimator. The other two approaches are new and are based on extensions of the approach proposed by Kiefer and Vogelsang (2002a). In Section 3, we derive the relevant asymptotic theory. We tabulate useful critical values. Additionally, we discuss asymptotic power of the tests in a special case. In Section 4, we use Monte Carlo simulations to examine the finite sample performance of the test statistics. We observe that the tests work best if the number of restrictions being tested is small relative to the sample size. Additionally, we find that the HAC-based tests have serious size distortions, while the new tests perform satisfactorily. In Section 5, we apply our tests to six European real per capita GDP series relative to Italy. In 1950 all six countries had levels of real per capita GDP higher than Italy. Our tests indicate that from 1950 to 1992 Italy has grown faster than those six countries although the null hypothesis that the relative rates are equal can be rejected. Our results suggest that Italy has been catching up to much of Europe on a systematic basis.

2. The model and test statistics

In this section we present the model, parameter estimation and the relevant test statistics.

[1] It should be noted that the asymptotic theory for multivariate time series regressions developed by Park and Phillips (1988) includes our model as a special case where no covariates are included. However, they do not consider hypotheses involving cross equation restrictions on the linear trend parameters as we do here.

2.1. Representation

Consider n trend-stationary time series denoted by $y_{1,t}$ to $y_{n,t}$ with $t = 1, 2, \ldots, T$, and assume that they can be represented by

$$y_{i,t} = \mu_i + \beta_i t + u_{i,t} \quad i = 1, \ldots, n. \tag{1}$$

Define the three $n \times 1$ vectors u_t, μ and β by $(u_{1,t}, u_{2,t}, \ldots, u_{n,t})'$, $(\mu_1, \mu_2, \ldots, \mu_n)'$ and $(\beta_1, \beta_2, \ldots, \beta_n)'$, respectively. It is assumed that a functional central limit theorem applies to u_t, that is,

$$T^{-1/2} \sum_{t=1}^{[rT]} u_t \Rightarrow \Lambda W_n(r), \tag{2}$$

where $\Rightarrow$ denotes weak convergence, $W_n(r)$ is an $n \times 1$ vector of standard independent Wiener processes, and $[rT]$ is the integer part of rT. See, for example, Phillips and Durlauf (1986) for conditions under which (2) holds. When u_t is covariance stationary it follows that Λ is the matrix square root of the matrix Ω defined as

$$\Omega = \Lambda \Lambda' = \sum_{j=-\infty}^{\infty} \Gamma_j,$$

where $\Gamma_j = E[u_t u_{t-j}']$. It is well known from the time series literature that Ω is equal to 2π times the zero-frequency spectral density matrix of the vector, u_t. Note that for a functional central limit theorem of the form given by (2) to hold, Ω is required to be finite and positive definite. This rules out stationary time series with long memory.[2]

2.2. Estimation

The parameters in (1) can be estimated by applying ordinary least-squares (OLS) equation by equation, which results in $\hat{\mu}$ and $\hat{\beta}$. If the errors are second-order stationary (a typical condition under which (2) will hold), then from the classic results of Grenander and Rosenblatt (1957), OLS is asymptotically equivalent to GLS (and equivalent to MLE under Gaussian errors). In addition, because (1) is a seemingly unrelated regression (SUR) with the same regressors in each equation, OLS is equivalent to the SUR estimator, which is the GLS estimator that accounts for contemporaneous correlation across the series. Thus, OLS has some nice optimality properties.

It will be convenient to express $\hat{\beta}_i$ as follows. Define $\bar{t} = T^{-1} \sum_{t=1}^{T} t$ and $\tilde{t} = t - \bar{t}$, then

$$\hat{\beta}_i = \left(\sum_{t=1}^{T} \tilde{t}^2 \right)^{-1} \sum_{t=1}^{T} \tilde{t} y_{i,t},$$

[2] In the case of stationary long memory processes, alternative functional central limit theorems could be used. However, the distribution theory obtained would differ substantially from what is obtained in this paper.