Comment on statistical analyses in "Identification of Aniakchak (Alaska) tephra in Greenland ice core" by N.J.G. Pearce et al.

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Pearce et al. [2004] compared tephra that was retrieved from Greenlandic ice (by *Hammer et al.* [2003]) with tephra from Aniakchak, Alaska. Largely on the basis of statistical analyses of the geochemical data, Pearce et al. claimed that the Greenlandic tephra matches with Aniakchak. Herein, those statistical analyses are examined.

Pearce et al. [2004] devote a section of their paper to discussing the "statistical distance" between two sets of tephrochronological data. The formula for statistical distance involves standard deviations. The term "standard deviation", though, can mean different things in different contexts: either the potential error of a measurement (i.e. precision) or the inter-particle dispersion (i.e. indicating variation among different particles). *Pearce et al.* [2004, table 2, caption] note this distinction when presenting their data. In applying the formula for statistical distance, however, they confuse the two meanings: the formula requires measurement precision, but they use inter-particle dispersion instead. This confusion makes the statistical-distance calculations of *Pearce et al.* [2004, sect. 6] erroneous. (There is no immediate way of fixing the calculations, because when the correct meaning of "standard deviation" is used, the D^2 statistic of Pearce et al. no longer has a chi-squared distribution.)

A second statistical problem with the work of *Pearce et al.* [2004] is in the use of "relative differences". The problem is easily seen via a hypothetical example: consider two tephras, both measured by two laboratories, with the following results for SiO_2 (on bulk samples, with measurement standard deviations indicated).

Lab 1 tephra 1: 50±0.6; tephra 2: 55±0.6
Lab 2 tephra 1: 50±3.0; tephra 2: 55±3.0

For Lab 1, the SiO_2 abundances do not overlap even at four standard deviations; so the lab has shown that the two tephras are different. For Lab 2, the SiO_2 abundances overlap at one standard deviation; so the lab has not shown that the two tephras are different. Yet both labs found the same relative difference of 10%. Thus, whichever way the relative difference of 10% is interpreted—to indicate dissimilarity of the tephras or not—the interpretation must be wrong for one of the labs. This problem arises because the relative-difference method fails to consider potential measurement errors. (Indeed, it is intuitively clear that such errors need to be considered when making comparisons.) Because of this, relative differences should not be used.

Thus both statistical analyses of *Pearce et al.* [2004] are incorrect. Hence the comparison of the tephras from Greenland and Aniakchak, presented by Pearce et al., is invalid. How should the tephras be compared? The only other multi-particle tephra comparison of which I am aware is that of *Keenan* [2003]. The comparison method used there, though, is not necessarily adequate for the situation here. Hence comparison of the two tephras requires further research.

References

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