Section 5.3

5.3 Plot the global temperature series, `gtemp`, and then test whether there is a unit root versus the alternative that the process is stationary using the three tests, DF, ADF, and PP, discussed in Example 5.3. Comment.

5.4 Plot the GNP series, `gnp`, and then test for a unit root against the alternative that the process is explosive. State your conclusion.

5.5 Verify (5.33).

Section 5.4

5.6 Investigate whether the quarterly growth rate of GNP exhibit ARCH behavior. If so, fit an appropriate model to the growth rate. The actual values are in `gnp`; also, see Example 3.38.

5.7 Weekly crude oil spot prices in dollars per barrel are in `oil`; see Problem 2.11 and Appendix R for more details. Investigate whether the growth rate of the weekly oil price exhibits GARCH behavior. If so, fit an appropriate model to the growth rate.

5.8 The `stats` package of R contains the daily closing prices of four major European stock indices; type `help(EuStockMarkets)` for details. Fit a GARCH model to the returns of one of these series and discuss your findings. (Note: The data set contains actual values, and not returns. Hence, the data must be transformed prior to the model fitting.)

5.9 The 2×1 gradient vector, $l^{(1)}(\alpha_0, \alpha_1)$, given for an ARCH(1) model was displayed in (5.47). Verify (5.47) and then use the result to calculate the 2×2 Hessian matrix

$$l^{(2)}(\alpha_0, \alpha_1) = \begin{pmatrix} \partial^2 l/\partial\alpha_0^2 & \partial^2 l/\partial\alpha_0\partial\alpha_1 \\ \partial^2 l/\partial\alpha_0\partial\alpha_1 & \partial^2 l/\partial\alpha_1^2 \end{pmatrix}.$$

Section 5.5

5.10 The sunspot data (`sunspotz`) are plotted in Chapter 4, Figure 4.31. From a time plot of the data, discuss why it is reasonable to fit a threshold model to the data, and then fit a threshold model.

Section 5.6

5.11 Let S_t represent the monthly sales data in `sales` ($n = 150$), and let L_t be the leading indicator in `lead`. Fit the regression model $\nabla S_t = \beta_0 + \beta_1 \nabla L_{t-3} + x_t$, where x_t is an ARMA process.